FLOW INSTABILITY OCCURRING IN THE COLLISION OF JETS OF AN IDEAL INCOMPRESSIBLE FLUID

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The collision of plane jets of an ideal incompressible fluid has been examined in detail in a number of investigations [1-4]. Figure 1 shows the resulting flow configuration in the coordinate system connected with the contact point, where OC and OD are jets colliding at the angle $\gamma = \gamma_1 + \gamma_2$; OA and OB represent the main flow, directed parallel to the Ox axis, and the reverse jet, making the angle χ with the Ox axis; O is the contact point. The interest in this problem stems from the fact that such a simple model satisfactorily describes the high-velocity oblique collision of metal plates, especially the formation of the reverse jet and the wave formation at the collision boundary of the plates. Meanwhile, the main parameters of the reverse jet are in good agreement with the results of calculations performed within the framework of an ideal incompressible fluid [1, 2].

Wave formation at the collision boundary of plates was discovered relatively recently [5]. The details of this phenomenon and many of the concepts advanced to explain it were examined in [6, 7]. In [8, 9], the opinion was ventured that the generation of waves is due to instability of the flow beyond the contact point. It was shown in [10-12] on the basis of both quantitative and qualitative estimates that the main laws of the wave generation process can be deduced by this approach. The stability of the symmetrical collision of jets was studied in [13] with allowance for the two-dimensional nature of the initial flow. It was shown that the jet configuration is unstable against symmetrical potential perturbations and stable against antisymmetrical perturbations. It was concluded on the basis of this that the waves seen in the high-velocity oblique collision of metal plates are vortical in nature.

The collision of jets of arbitrary thickness has yet to be analyzed with regard to stability. Yet this problem is quite important, especially in light of the indeterminate nature of the problem of the collision of plane jets of an ideal incompressible fluid. For example, conservation laws do not permit the determination of the angle χ, which specifies the direction of the reverse jet. Thus, an investigation of stability is needed to determine stable jet configurations, should such configurations exist in general. The solution of this problem is the goal of the present investigation.

Formulation of the Problem. We assume that the flow which develops in the collision of plane jets of an ideal incompressible fluid is a potential flow. Then the solution describing the collision of two flows of the thickness h_1 and h_2 having equal velocities at infinity has the form [3]

$$\pi w = \ln(1 + v/a_1) + h_2 \ln(1 + v/a_2) - k_1 \ln(1 + v) - k_2 \ln(1 - v/a_3), \quad (1)$$

where $v = v_x - iv_y$ and $w = \varphi + i\psi$ are the complex velocity and potential of the flow; k_1 and k_2 are the thicknesses of the main flow and the reverse jet; $a_1 = e^{-i\gamma_1}$; $a_2 = e^{i\gamma_2}$; $a_3 = e^{-i\varkappa}$. All of the quantities are dimensionless. As the units of velocity and length, we respectively chose the velocity of the colliding flows at infinity and the thickness h_1. The densities of the jets were assumed to be identical.

To determine the four unknowns – k_1, k_2, χ, and γ_1 or γ_2 (the total collision angle γ is given), we have three equations which follow from the mass and momentum conservation laws. Thus, solution (1) has one indeterminate parameter. It is convenient to choose the angle χ as this parameter, in which case

$$k_1 = 1 + h_2 - k_2,\ k_2 = [1 - \cos\gamma_1 + h_2(1 - \cos\gamma_2)]/(1 + \cos\chi), \quad (2)$$

while the connection between the angles γ_1 and γ_2 is found from the equation

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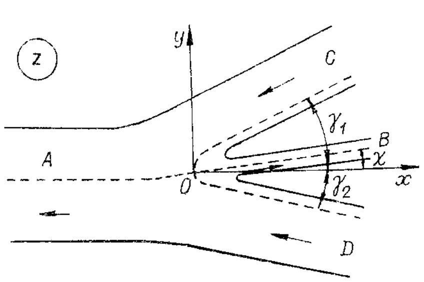

Fig. 1

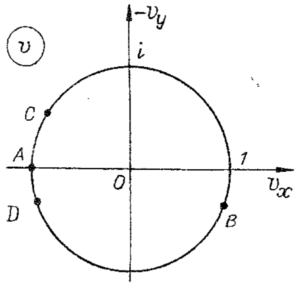

Fig. 2

$$(1 - \cos\gamma_1 + h_2(1 - \cos\gamma_2))\,\mathrm{tg}\,\chi = h_2 \sin\gamma_2 - \sin\gamma_1, \tag{3}$$

i.e., all of the constants in Eq. (1) are assumed to be known.

We will use the method in [14, 15] to analyze the stability of the jet configuration. We assumed that, as the main flow, the disturbed flow is also a potential flow. The complex potential of small perturbations will be designated as $w + w_1$. Two conditions must be satisfied on the free surface: 1) pressure is equal to zero (dynamic condition); 2) particles originally located on the free surface remain on it at subsequent moments of time (kinematic condition). It is convenient to subsequently perform the study in the plane of complex velocity v. The flow shown in Fig. 1 is reflected on the interior of a circle of the radius 1 (Fig. 2) [3]. The conditions on the free surface, which correspond to the circle $vv^* = 1$ (the asterisk denoting the operation of complex conjugation), lead to an equation for w_1 [15]:

$$\mathrm{Im}\left\{D\left[w_1 - v\frac{dw}{dv}D[w_1]\right] - \frac{\partial w_1}{\partial t}\right\} = 0, \tag{4}$$

where D is the differential operator $\partial/\partial w + \partial/\partial t$; t is dimensionless time. Designating the expression in braces as H(v) and considering v to be the independent variable, we obtain

$$H(v) = (H(v))^* \text{ at } vv^* = 1. \tag{5}$$

Expansion of Eq. (4) gives us the boundary condition for the potential φ_1, which was studied in [16] for several problems. Here, we analyze Eq. (5) directly.

We seek the dependence of the solution on time in the form [15]

$$w_1 = G_1(v)\,e^{\omega t} + G_2(v)\,e^{\omega^* t}. \tag{6}$$

The allowable values of ω and the corresponding functions G_1 and G_2 are found from (5). The general solution can be represented as the sum of functions of the form (6). Perturbations for which $\mathrm{Re}\{\omega\} > 0$ will be unstable. After insertion of (6) into (5), the conditions on the free surface lead to the equation

$$L_\omega[G_1(v)] = (L_{\omega^*}[G_2(v)])^*, \tag{7}$$

where

$$L_\omega[G] = \frac{v^2}{\Omega}\frac{d^2G}{dv^2} + 2\omega v\frac{dG}{dv} + \omega\left(\frac{d\Omega}{dv}\frac{v}{\Omega} + \omega\Omega\right)G,$$
$$\Omega = \frac{v}{\pi}\left(\frac{1}{v+a_1} + \frac{h_2}{v+a_2} - \frac{k_1}{v+1} - \frac{k_2}{v-a_3}\right). \tag{8}$$

If the flow is symmetrical relative to the Ox axis, then it is easy to establish a connection between the functions G_1 and G_2 [15]. This makes it possible to obtain the boundary condition for one function G_1, which was studied in [13] (for example) in an analysis of the stability of a symmetrical jet configuration. This method cannot be used in the case of colliding flows of arbitrary thickness. We therefore examine a special case of Eq. (7), to wit: let

$$L_\omega[G_1(v)] = C_0; \tag{9}$$

$$L_{\omega^*}[G_2(v)] = C_0 \tag{10}$$

(C_0 is a real constant). Equations (9) and (10) must be satisfied with $vv^* = 1$. Using the property of analytic continuation, we assume that these equations are valid over the entire plane v. Thus, the problem is reduced to finding the functions $G_1(v)$ and $G_2(v)$ which satisfy inhomogenous second-order differential equations (9) and (10). In all subsequent relations, ω is encountered in the form of the combination ω/π. For brevity, this will be designated simply as ω.

Let us study Eqs. (9) and (10), which are inhomogeneous linear second-order differential equations of the Fuchs type [17] with regular singular points 0, −1, $-a_1$, $-a_2$, $-a_3$ and ∞. If $K^{(1)}$ and $K^{(2)}$ are two linearly independent solutions of the corresponding homogeneous equation, then the general solution has the form [17]

$$G_1(v) = A_1K^{(1)} + B_1K^{(2)} + K^{(2)}\int_0^v \frac{K^{(1)}P}{W}dv - K^{(1)}\int_0^v \frac{K^{(2)}P}{W}dv, \tag{11}$$

where $P = C_0\Omega/v^2$; W(v) is a Wronskian which can be found from the differential equation and is equal to

$$W(0)(1+v)^{2\omega k_1}(1-v/a_3)^{2\omega k_2}(1+v/a_2)^{-2\omega k_2}(1+v/a_1)^{-2\omega}.$$

The function $G_2(v)$ will obviously be given by Eq. (11) if we replace ω by ω* and A_1 and B_1 by A_2 and B_2. The solution of (1) satisfies the necessary conditions everywhere on the free surface except for the singular points. In the neighborhood of the singular points, the behavior of the functions G_1 and G_2 should satisfy certain physical requirements.

We will study the solution at $v \to -1$. In the z plane, this corresponds to motion downstream in the main flow, where the jet asymptotically becomes a straight line. Here, velocity is constant. It is known [18] that such a flow is neutrally stable and that any perturbation is propagated without a downstream change in the velocity of the jet. This fact can be expressed mathematically by equating the total derivative of perturbation rate with respect to time to zero. This leads to the requirement of the existence of the limits

$$\lim_{v\to-1}(v+1)^{-\omega k_1}G_1 \text{ and } \lim_{v\to-1}(v+1)^{-\omega^* k_1}G_2.$$

Let us analyze the function G_1. The two linearly independent solutions of homogeneous equation (9) $K^{(1)}$ and $K^{(2)}$ in the neighborhood of the point −1 behave as $(v+1)^{\omega k_1+1}$ and $(v+1)^{\omega k_1} - \omega k_1(v+1)^{\omega k_1+1}\ln(v+1)$, i.e., the necessary conditions are satisfied automatically. Studying the particular solution of the inhomogenous equation, we find that the boundary conditions are satisfied if

$$\mathrm{Re}\{\omega\} \leqslant 1/k_1. \tag{12}$$

We obtain the same inequality for G_2. The restriction on the solution in the neighborhood of the singular point $v = a_3$ is found in a similar manner (with the replacement of k_1 by k_2). However, it can be assumed that $k_2 \le k_1$, and no additional restrictions are imposed on ω.

At $v \to -a_1$ and $v \to -a_2$, the functions G_1 and G_2 should vanish, since the colliding jets are not disturbed at infinity. Let us study G_1. The solutions of homogenous equation (9) have the following asymptote at $v \to -a_1$

$$K^{(i)} \sim \alpha_{1i}(v+a_1)^{-\omega+1} + \beta_{1i}\left[(v+a_1)^{-\omega} + \omega a_1^*(v+a_1)^{-\omega+1}\ln(v+a_1)\right]. \tag{13}$$

Here, α_{1i} and β_{1i} (i = 1.2) are known constants − functions of a_1, a_2, a_3, and ω.

The character of the behavior of the particular solution at $v \to -a_1$ depends on ω. We are interested in solutions with a positive real part ω. However, when Re{ω} > 0, the integrals in (11) converge at $v \to -a_1$. We will designate their values through C_0J_{11} and C_0J_{12}. In this case, obviously, the asymptote of the particular solution will be the same as with $K^{(1)}$ and $K^{(2)}$. This means that in order for the function G_1 to vanish at $v \to -a_1$ and Re{ω} > 0, it is necessary to equate the coefficient with $(v+a_1)^{-\omega}$ in (11) to zero. This leads to the equality

$$\beta_{11}A_1 + \beta_{12}B_1 + (\beta_{12}J_{11} - \beta_{11}J_{12})C_0 = 0, \tag{14}$$

which links the constants A_1, B_1, C_0. We study the singular point $-a_2$ in a similar manner. The function G_1 approaches zero at $v \to -a_2$ and Re{ω} > 0 if the following equality is satisfied

$$\beta_{13}A_1 + \beta_{14}B_1 + (\beta_{14}J_{13} - \beta_{13}J_{14})C_0 = 0, \qquad (15)$$

where β_{13} and β_{14} are constants of the expansion of $K^{(1)}$ and $K^{(2)}$ in the neighborhood of $-a_2$, analogous to β_{11} and β_{12} [this expansion differs from (13) by the replacement of a_1 by a_2 and ω by ωh_2]; J_{13} and J_{14} are values of the integrals in (11) with $v = -a_2$.

We also find two equations for the functions G_2: the first follows from the requirement that G_2 vanish at $v \to -a_1$, while the second follows from the requirement that G_2 vanish at $v \to -a_2$:

$$\beta_{21}A_2 + \beta_{22}B_2 + (\beta_{22}J_{21} - \beta_{21}J_{22})C_0 = 0; \qquad (16)$$

$$\beta_{23}A_2 + \beta_{24}B_2 + (\beta_{24}J_{23} - \beta_{23}J_{24})C_0 = 0. \qquad (17)$$

The constants β_{2i} and J_{2i} ($i = 1, 4$) have the same significance as the corresponding quantities in (14) and (15), differing only in the fact that they depend on ω^* rather than ω.

We obtained two systems (14)-(15) and (16)-(17) to determine A_1, B_1 and A_2, B_2 through C_0 (the constant C_0 remains arbitrary and is found from the initial conditions). These systems have a unique solution when their determinant is nontrivial. For example, if the determinant of the first system is equal to zero, then assuming that $C_0 = 0$ and $G_2 \equiv 0$, we will have one equation linking A_1 and B_1, i.e., all of the boundary conditions can be satisfied by choosing the general solution of homogeneous equation (9) as the general solution and assuming that $w_1 = G_1e^{\omega t}$. We proceed in the same manner if the determinant of the second system is equal to zero. Then $w_1 = G_2e^{\omega^* t}$. If both determinants are simultaneously equal to zero, then $G_1e^{\omega t}$ and $G_2e^{\omega^* t}$ are the solutions of the problem. In any case, it turns out to be possible to equate the coefficients with the divergent terms $(v + a_1)^{-\omega}$ and $(v + a_2)^{-\omega h_2}$ to zero when $\mathrm{Re}\{\omega\} > 0$. Taking (12) into account, we finally obtain the following result: the jet configuration is unstable against small potential perturbations; the boundary conditions are satisfied with positive values of the real part ω, which lies within the interval

$$0 < \mathrm{Re}\{\omega\} \leqslant 1/k_1. \qquad (18)$$

The symmetrical collision of flows was studied in detail with regard to stability in [13]. It was shown there that the random configuration is unstable against symmetrical perturbations. Meanwhile, an inequality that coincides with (18) was found for the real part of ω. Thus, the perturbations in relation to which the arbitrary jet configuration is unstable become symmetrical at $h_2 = 1$ and $\chi = 0$.

Equations (9) and (10) are readily solved in finite form at $\omega = 0$. The resulting function w_1 satisfies all boundary conditions. Thus, $\omega = 0$ is the eigenvalue of the problem.

In the above study of the stability of a jet configuration, we assumed that the disturbed flow is a potential flow. The class of allowable perturbations is severaly limited. However, even relative to this limited class, the problem of the collision of jets of arbitrary thickness is unstable. It would be correct to use approaches in which it is assumed that waves are formed at the collision boundary between metals as a result of instability of the flow beyond the contact point (see, e.g., [10-12]).

Another consequence of the hydrodynamic stability which occurs is decay of the reverse jet. The basis for this conclusion is the well-known empirical fact that stable reverse jets are not seen in the wave-formation regime. Instead, a cloud of atomized particles is observed [6, 7]. Since waves develop in the main flow due to instability of the initial flow, it is natural to expect that the occurrence of this instability in the reverse jet will lead to its disintegration.

Finally, as already noted, the problem of the collision of jets in the general case is indeterminate. The author of [4] formulated several factors which if taken into consideration could either eliminate or restrict this indeterminateness. In particular, it was indicated that it is necessary to determine the stable jet configurations. The results obtained in the present study show that such configurations do not exist. It is interesting to note that, in accordance with (18), the real part of ω takes on minimal positive values when the thickness of the main flow is maximal. Using Eqs. (2) and (3), we can show that k_1 reaches a maximum at $\chi = 0$. It was this condition, obtained from completely different considerations, that was proposed in [19] to close system (2)-(3). Practical interest in the problem continues to stimulate searches for a closing relation. Two examples of this, different from [19], can be found in [20, 21]. The author of [20] examined the position of the centers of inertia of fluid particles in colliding and diverging flows and obtained the condition $\gamma_1 = \gamma_2 + \chi$ (to use the notation employed in the present study). In [21], the following hypothesis was formulated on the basis of analysis of empirical data: it is possible to realize a configuration in which the zero streamline in the outgoing jets has minimal curvature. No single configuration has an advantage over another from the viewpoint of stability, however, since all configurations are unstable within the framework of an ideal incompressible fluid.

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